ANALYSIS OF BUBBLE-CHAMBER EVENTS
CONTAINING NEUTRAL PIONS

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In this report we wish to consider the problem of analyzing events

which have one π^o in them. Several papers have pointed out the desira-

bility of measuring charged outgoing particles with sufficient accuracy

to determine the presence of missing neutrals. What we wish to analyze

is the corresponding question when there is at least one π^o present.

The type of question we wish to answer: in a four-prong event can I

distinguish $\pi^- p \to p\pi^+ \pi^- \pi^- \pi^o$ from $\pi^- p \to \pi^+ p\pi^- \pi^- \pi^o$ i.e., can I tell which

positive track is the proton? Similarly, can I distinguish $\pi^- p \to pk^+ k^- \pi^- \pi^o$

from $p\pi^+ \pi^- \pi^- \pi^o$? In events with another neutral beyond the π^o can I ob-

tain a measure of the mass of the missing neutral $\pi^- p \to p\pi^- \pi^o + MN$?

We shall confine our analysis to the problem of energetic π^o's

roughly one GeV or higher because in the relativistic limit we can make

simple approximations for the kinematics of the pion. The problem of

low-energy π^o's appears more difficult and will have to be handled with

a more detailed analysis.

In Section I we shall make a rough comparison of charged particle

and π^o detection. We shall review the results of the accuracy of deter-

mination for charged particles available from several detectors and

compare this to π^o accuracies. In Section II, we shall present the re-

sults for the determination of events with π^o's present. In Section III,

we will summarize briefly the properties of some of the π^o detection

systems and comment on complications in our simplified analysis of

Section I. Finally, in Section IV we would like to outline some impor-

tant problems in detection which we have not considered in the hope

that some of these questions may be answered at a later time.

Throughout this report we have drawn principally on the analysis

of Trilling (SLAC 5) for the determination of π^o's and there most of

the formulas which we need are derived.

Section I--Comparison of π^o and Charged-Particle Detection Accuracies

We will wish to consider the accuracy of determination of the

longitudinal momentum and the transverse momentum for energetic

outgoing tracks. It will appear that a useful measure of the accuracy

of π^o's is $\Delta p/p$ for longitudinal momentum and $\Delta p_t/m_o$ for transverse

momentum where m_o is the mass of the π^o. The accuracy obtainable

for charged-particle tracks in large bubble chambers has been con-

sidered by Kramer and Derrick, NAL Summer Study Report A. 1-68-35,

for hybrid spectrometers by Fields et al., NAL Summer Study Report

A. 3-68-12. In the former report, Fig. 4 gives the accuracy of the

$\Delta p/p$. To estimate the accuracy of transverse momentum we have used

an estimate for the angular accuracy of $4\epsilon/\ell$ where ϵ is the setting

error and ℓ is the length of the track. For the corresponding figure

of the hybrid spectrometer we have used the figure of 0.1 GeV uncer-

tainty in longitudinal momentum at high momentum, and we have

estimated as the angular accuracy $2\epsilon/\ell$. This latter estimate improves

the accuracy of the transverse momentum substantially: we feel a

more detailed estimate including multiple scattering should be made

but our estimate is probably more realistic than the figure given by

Fields et al. The results are given in Table I.

Table I. $\Delta p/p$ for Charged Particles.

Device	1 GeV/c	5 GeV/c	10 GeV/c	20 GeV/c	100 GeV/c
25-ft chamber[a]	0.0017		0.0017		0.0023
12-ft chamber 40 kG[a]	0.0023		0.0023		0.005
12-ft chamber 20 kG[a]	0.0048		0.0048		0.010
Hybrid	0.007	0.013	0.0015	0.003	0.001

[a] Taken from Fig. 4, NAL Summer Study Report A. 1-68-35.

Table II. $\Delta p_t/m_o$ for Charged Particles.

Device/Momentum	1 GeV/c	5 GeV/c	10 GeV/c	20 GeV/c	100 GeV/c
25-ft chamber	0.002	0.011	0.022	0.044	0.22
12-ft chamber 40 kG[a]					
12-ft chamber 20 kG					
Hybrid[b]	0.005	0.025	0.050	0.094	0.155

[a] Using $\Delta p_t \approx 4(\epsilon/\ell)p$ with values of Fig. 4, NAL Summer Study Report A. 1-68-35.
[b] From NAL Summer Study Report A. 3-68-12, we feel that the multiple-scattering contribution can be reduced using a thinner exit window. A streamer chamber would also reduce the contribution.

In order to obtain similar estimates for π^o's we will consider two

situations, the first in which only the opening angle of the two γ rays is

available, the second in which additional information on the momenta of

the two γ-rays is available. We will use the formulas given in Appendix

I by Trilling and repeat his notation. First, we introduce appropriate

symbols as follows:

β, p, E   =   velocity, momentum, total energy of incoming pion

θ =   opening angle between the two photon directions

ξ =   angle between π^o direction and the line bisecting

                                 the angle θ

p_1, p_2 =   momenta of the two photons

θ^* =   angle between π^o direction and direction of photon

                                 #1 in the π^o rest system

p_{1m}, p_{2m} =   measured momenta of the photons

Δp_1, Δp_2 =   rms errors in the measured values of the photon

                                 momenta

m   = π^o mass

the opening angle θ is then given by

$$\cot \theta / 2 = p / m \sin \theta^*.$$

We see from this that a lack of knowledge of θ^*, when we mea-

sure the opening angle only, will produce a large uncertainty in β.

Further, any uncertainty in measuring θ will produce a further un-

certainty in p. The former can, of course, be overcome by obtaining

information on p_1 and p_2, the momenta of the two photons, the latter

presumably is characteristic of the detection system. Since the

opening angle θ gets very small at high p (for 10 GeV π^o the minimum

opening angle is 1.5°) there are difficulties in measuring this angle accurately. Also, because the spatial separation of the showers will be small, there may be difficulties in measuring the momenta of the two photons separately. (At 1 m from the sources the two photons would be only 2.7-cm apart). We, therefore, consider the two sources of error in obtaining an estimate of the error. We define $\Delta p/p$ for this case (no knowledge of p_1 or p_2) as $(p_{calc.} - p)/p$ where $p_{calc.}$ is the value calculated for p assuming $\theta^* = 90°$.

The contribution from θ^* is then

$$\left.\frac{\Delta p}{p}\right|_{\theta^*} \simeq 1 - \sin\theta^*, \tag{1}$$

using $p_{calc.} \simeq \frac{2m}{\theta}$, $p\sin\theta^* \simeq \frac{2m}{\theta}$. If we assume the separation of the γ conversion points is measured with an accuracy 2ϵ and that we are a distance L downstream from the π^o production point then the contribution from measuring uncertainty in θ is

$$\left.\frac{\Delta p}{p}\right|_{\theta} \simeq \frac{p}{2m}\frac{2\epsilon}{L} \simeq \frac{p}{m}\frac{\epsilon}{L} . \tag{2}$$

There are similar contributions to the error in the transverse momentum of the π^o if only the opening angle is measured. The contributions again come from our lack of knowledge of θ^* and the uncertainty in measuring angles. The useful formula is given by Trilling (Eq. 2)

$$\sin\xi = \cos\theta^* \sin\frac{\theta}{2}, \tag{3}$$

or for high π^o momenta

$$\xi \simeq \frac{\theta}{2} \cos \theta^*.$$ (4)

Defining as the uncertainty in the transverse momentum Δp_t the difference between the momentum along the bisector of the two γ rays and the true momentum we have

$$\frac{\Delta p_t}{m} \bigg|_{\theta^*} \simeq \frac{p_{calc.}}{m} \xi = \frac{2m}{\theta m} \frac{\theta}{2} \cos \theta^*$$ (5)

$$= \cos \theta^*.$$

The uncertainty in Δp_t because of the uncertainty in measuring angles is just

$$\frac{\Delta p_t}{m} \bigg|_{\theta} = \frac{p}{m} \frac{\epsilon}{L}.$$ (6)

From formulas (2) and (6) we see that uncertainties in measuring angles contribute roughly $\frac{p}{m} \frac{\epsilon}{L}$ to $\frac{\Delta p_1}{p}$ and $\frac{\Delta p_t}{m}$, that is the contribution to momentum uncertainties grow with the π^o momentum and the only way to keep these contributions small is to have very large L for high momenta. The contributions from lack of knowledge of θ^* can be reduced with information from the momenta of the individual γ rays.

When information from the individual γ rays is available, Trilling has shown that a least-square approach yields (Trilling Eq. 18)

$$\frac{\Delta p}{p} = \frac{\cos \theta^*}{\sqrt{\left(\frac{p_1}{\Delta p_1}\right)^2 + \left(\frac{p_2}{\Delta p_2}\right)^2}} \simeq \frac{\Delta p_\gamma}{p_\gamma} \frac{\cos \theta^*}{\sqrt{2}} \quad (7)$$

where $\Delta p_\gamma / p_\gamma$ is the uncertainty in measuring the γ-ray momentum averaged for the two γ rays. Similarly, using Trilling (17),

$$\frac{\Delta p_t}{m} \simeq \frac{p \Delta \xi}{m} = \frac{\sin \theta^*}{\left(\frac{p_1}{\Delta p_1}\right)^2 + \left(\frac{p_2}{\Delta p_2}\right)^2}$$

$$= \left(\frac{\Delta p_\gamma}{p_\gamma}\right) \frac{\sin \theta^*}{\sqrt{2}} \quad . \quad (8)$$

We see that the longitudinal and transverse momentum uncertainties measured in terms of the momentum and π^o rest mass respectively are to roughly one-half the fractional error of γ-ray energy determination. Of course, this holds only as long as the opening angle determination are small compared to this, i.e., as long as $\frac{p}{m} \frac{\epsilon}{\ell}$

$< \frac{\Delta p_\gamma}{2 p_\gamma}$ otherwise the angle determination uncertainty dominates. If we assume $\Delta p_\gamma / p_\gamma \approx 0.2$ and for a large bubble chamber ϵ / ℓ

$\approx 500 \mu / 1 m = 0.5 10^{-3}$ this implies that for $p < 27$ BeV, $\Delta p_\gamma / p_\gamma$ dominates, while for $p > 27$ BeV, angle errors dominate. Assuming that

$\Delta p_\gamma / p_\gamma \approx 0.2$, we see $\Delta p / p \sim 0.1$ and, therefore, is much larger than the values given in Table I for charged particles. For $\Delta p_t / m_o$,

a value of 0.1 for the π^o's is again larger than those for charged

particles given in Table II, at 100 GeV/c for bubble chambers the angle

uncertainty dominates both the charged and π^o errors in $\Delta p_t/m$, being

$\frac{p}{m}\frac{4\epsilon}{\ell}$ for charged particles as against $\frac{p}{m}\frac{\epsilon}{\ell}$ for π^o's so in this

domain the π^o transverse momentum can be more accurately determined

because no curvature problem is involved, however, as mentioned above

$\Delta p_\gamma/p_\gamma$ may be extremely difficult to determine.

We have unfortunately not had the time to handle the problem of

low-energy π^o's, nor the question of what accuracies on π^o determina-

tion are available when the total energy and the opening angles are

available.

Section II--Analysis Errors in π^o Events

In this section we wish to reconsider briefly some of the questions

concerned with analysis of events involving one π^o. For simplicity we

will assume that $\Delta p/p$ and $\Delta p_t/m$ both are 0.1 and that the π^o uncertain-

ties dominate the charged particle uncertainty. Then we can make

rough estimates, a) whether we can distinguish K^+K^- from $\pi^+\pi^-$ on a

pair of tracks, b) whether we can distinguish $K^+\pi^+$ from the interchange

π^+K^+ , c) how well we can determine the missing mass in a reaction

with one π^o. Trilling has shown that the distinction in cases a) and b)

can be estimated on the basis of the uncertainty in energy balance

using the constraint of conservation of longitudinal momentum. As-

suming with our results above that the error is dominated by the π^o

we find two terms, ΔE_1 from the uncertainty in longitudinal momentum

and ΔE_2 from the uncertainty in transverse momentum

$$\Delta E_1 = \left(\frac{\Delta p}{p}\right) \frac{p_t^2 - m^2}{2p} , \qquad (9)$$

and

$$\Delta E_2 = \frac{p_t}{p} \frac{\Delta p_t}{m} m , \qquad (10)$$

where we have used

$$p_t \Delta\theta = \frac{p_t}{p} \frac{p\Delta\theta}{m} m \simeq \frac{p_t}{p} \frac{\Delta p_t}{m} m . \qquad (11)$$

For our case outlined above we can estimate values of ΔE_1 and ΔE_2 for reasonable values of p_t and p. The difficulty in assessing the usefulness of these estimates comes from the fact that there will be wide variations in p_t and p, and it is only a rough estimate that we obtain. We must look in detail for specific models of π^o distributions to obtain reasonable distributions for ΔE_1 and ΔE_2. Our values should be considered as samples. It is clear that for small p_t and large p the parameters ΔE_1 and ΔE_2 decrease (until $\Delta p/p$ increases with p).

Table III. Energy Error ΔE. From Uncertainty
In Longitudinal Momentum, With $\Delta p/p = 0.1$

p_t \ p	1 GeV/c	10 GeV/c
0.5	0.012	0.001
0.25	0.002	0.0002

If the single π^o has small transverse momentum or large longitudinal momentum, its contribution no longer dominates the error ΔE_1, and more careful calculations are required. Similarly, for ΔE_2 using $\Delta p_t/m = 0.1$.

Table IV. Energy Error ΔE_2, From Uncertainty In Transverse Momentum.

	1 GeV/c	10 GeV/c
0.5	0.0067	0.0007
0.25	0.0033	0.0003

Again for small p_t of the π^o the contributions of charged particles may be important in ΔE_2.

II (a). Distinction between $K^+ K^-$ is given by

$$\Delta E' = \left(m^2_K - m^2\right)\left(\frac{1}{2p_1} + \frac{1}{2p_2}\right) \tag{9}$$

where p_1 and p_2 are the momenta of the two tracks. Hence, we find approximately

$$\Delta E' \simeq \frac{0.23}{\bar{p}} . \tag{10}$$

Therefore, unless the average momentum of the $\pi^+ \pi^-$ or $K^+ K^-$ pair is ten times larger than the π^o momentum, we can still make this distinction, since

$$\Delta E' > \sqrt{(\Delta E_1)^2 + (\Delta E_2)^2} \tag{11}$$

and we still have a significant energy unbalance for the wrong choice.

II(b). Distinction of mass interchange for two tracks.

When two non-identical mass particles have nearly the same momentum it becomes nearly impossible to distinguish which is which. Trilling (p. 48) shows that the effect of interchanging masses on the energy unbalance is

$$\Delta E'' = \frac{m_1^2 - m_2^2}{2} \frac{p_2 - p_1}{p_1 p_2} . \tag{12}$$

For $K\pi$ interchange using $(p_2 - p_1)/p_1 = 0.1$, we find

$$\Delta E'' = \frac{0.23}{2} \frac{0.1}{p} = \frac{0.011}{p} . \tag{13}$$

Comparing with Table III and IV we see that when the average momentum of the pair is larger than the π^0 momentum this distinction is not possible. The $K\pi$ must differ by 0.3 to allow the distinction to be made unless the π^0 momentum is much larger. We must add of course that even in the absence of π^0's this distinction is difficult.

II(c). Determination of masses of missing neutrals.

In this part we wish to consider the accuracy available for determining a missing neutral if one π^0 is seen, for example,

$$\pi^- p \rightarrow \pi^- p \pi^0 \text{ (MN)}. \tag{14}$$

In NAL A. 3-68-12 the error in the mass squared of the missing neutral is divided into two contributions: from the longitudinal and transverse momentum. In order to avoid confusion with our notation

let us call E (MN), $p_{\parallel}$ (MN), and p_t (MN) the energy, the parallel component of momentum and transverse components of momentum for the missing neutral. Then the contribution from the longitudinal momentum (again assuming the π^o dominates) is

$$dMM^2_p = 2 [- E(MN)\beta + p_{\parallel}(MN) \cos \theta - p_{\perp}(MN) \sin \theta]\Delta p, \qquad (15)$$

assuming near cancellation of $E(MN)$ and $p_{\parallel}(MN)$ and that β and $\cos \theta$ are also roughly the same for the π^o. A rough estimate for dMM^2_p is

$$dMM^2_p \simeq 2p_{\perp}(MN) p_t \left(\frac{\Delta p}{p} \right) . \qquad (16)$$

Using 0.5 for the transverse momenta and $\Delta p/p = 0.1$, we find $dMM^2 = 0.05$. We can estimate the contribution from the transverse momentum uncertainty of the π^o

$$dMM^2 \bigg|_{p_t} = -2 [-p_{\parallel}(MN) \sin \theta - p_{\perp}(MN) \cos \theta] \Delta p_t . \qquad (17)$$

Again assuming roughly $p \approx p_{\parallel}(MN)$ we can approximate

$$dMM^2_{p_t} \simeq -2p_{\perp}M \frac{\Delta p_t}{M}$$

$$= -2 \cdot 5 \cdot 135 \cdot 1$$

$$= 0.0135 .$$

We see that we have a rough estimate of 0.06 for dMM^2. Since

the MM^2 of the π^O, $2\pi^O$, η^O are 0.02, 0.09, 0.25 respectively we see

that we cannot make a distinction between one or two extra π^O but that

a missing η can be distinguished. We have been forced to make dras-

tic approximations in these calculations; the results should be taken

as rough indication until a complete analysis can be made.

Section III--π^O Detection Systems

Several systems for detecting π^O mesons can be considered. In

particular, the possibility of H_2-neon mixtures in a large bubble

chamber has been treated by G. Kalmus in UCRL-16830. Our Fig. 1

summarizes his results for 5 and 20 BeV/c electrons in three pos-

sible mixtures varying in radiation length from 50 to 250 cm. We give

the uncertainty in the electron momentum determination as a function

of the track length of the electron. The results certainly indicate

that in such a detector the uncertainty in the γ-ray energy (coming

from two electrons) should be less than 20% for a wide range of γ-ray

energies.

Trilling in his report in SLAC 5 has evaluated a system of plates

in hydrogen. We have evaluated his formulae and find again that for

a series of plates 20% uncertainty is available if a 2-m array similar

to the one he describes is used.

There are some obvious problems with the above systems which

should be investigated in detail. For the neon-H_2 combination it would

be desirable to have an inner interaction region of hydrogen particularly

if one wants to examine peripheral interactions where the neon back-

ground may be very serious. In both hydrogen-neon and plate arrays

we expect a serious number of secondary interactions from charged

particles which will make the identification of the source of γ rays

difficult. Unfortunately, we have not had the time to study the effect

quantitatively; however, some events in FAKE have been run which

when analyzed should give some answer to this question. From this

point of view a chamber (bubble or streamer) with thin walls and al-

lowing detection of γ rays with high efficiency outside the main volume

appears attractive. As mentioned above, for very energetic π^o's the

converters must be far from the chamber if the two showers are to

be physically separated. Another possibility is to consider the case

where the opening angle is determined with some precision, but only

the total energy of the π^o is determined. This would give rise to a

new set of criteria for the accuracy of the momentum determinations.

Throughout we have neglected the important problem of detection

of low-energy π^o's. These should be examined in a FAKE-type pro-

gram. In general, it appears that if the π^o detectors are outside the

main interaction volume, thin plate arrays offer an interesting possi-

bility. Such thin plates should cover a large fraction of the solid angle.

Section IV--Other Questions in γ Detection

In this report we have indicated that for events with one π^o

present the uncertainties in measurements of the π^o will be larger

than those of the charged particles for several detector systems.

Nevertheless, if a fractional uncertainty in γ-ray momentum of 0.2

can be achieved we have shown in Section II that a determination of

masses for outgoing charged tracks can be made. In this analysis

we have used only conservation of longitudinal momentum. The results

should be checked with detailed fitting. Missing mass determinations

are still possible but the errors are found to be rather large. Again

the rough results should be checked.

We have not considered the problem of two π^o's and how well one

can distinguish which γ rays should be paired. It appears that for in-

variant masses of 2 π^o's large compared to several π masses the dis-

tinction should be possible but that for smaller invariant masses the

γ rays will be confused. Again this problem should be investigated

statistically with a computer. In general the two γ decays of other

particles like the η_o should be easily distinguished from the π^o as

soon as some momentum information is available.

We conclude that it is very desirable at high energy to detect

the γ rays with high efficiency to allow the exploration of reactions

with one or more π^o's. We have only outlined some of the questions

which must be answered more precisely to design a good working system.
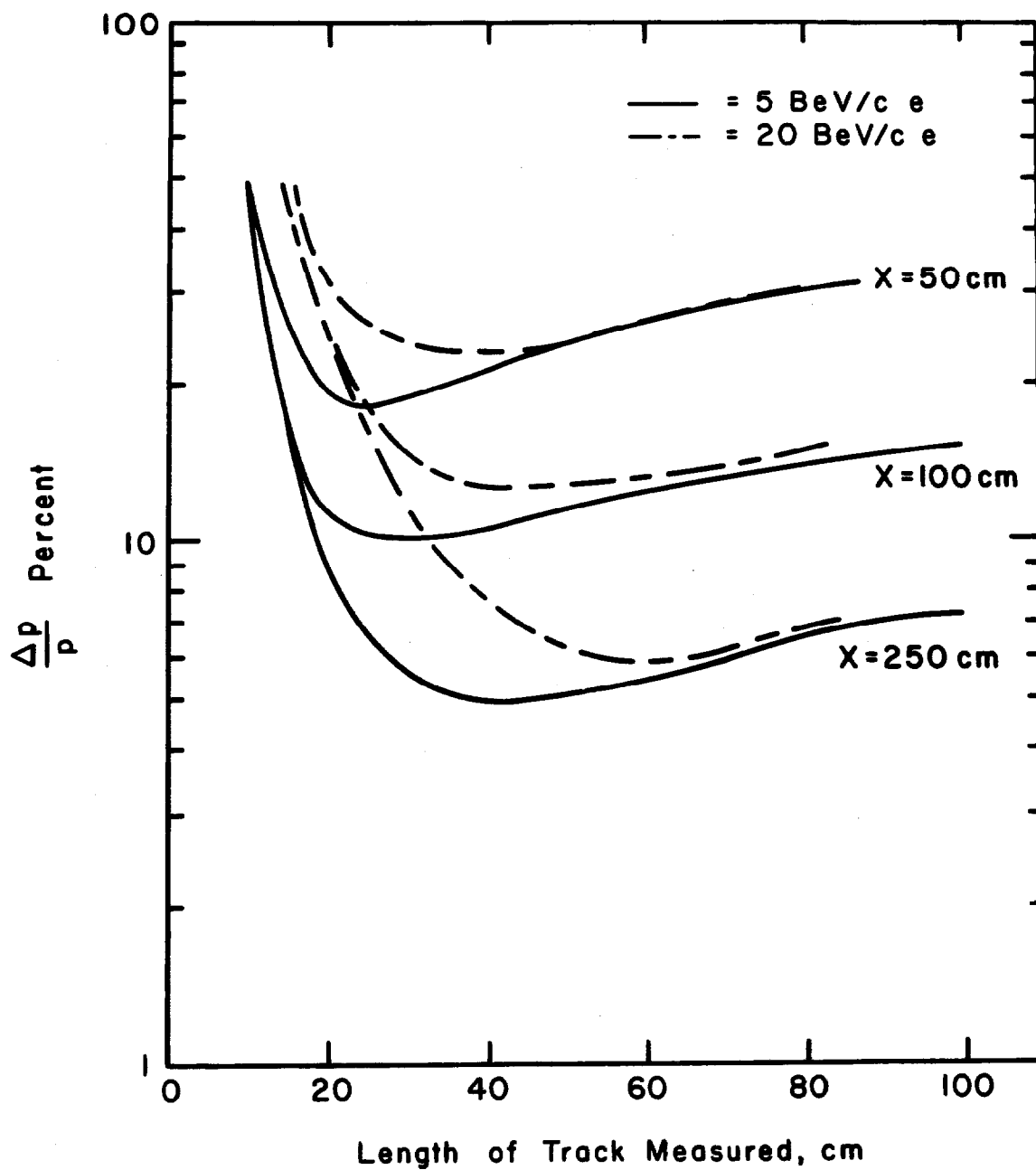
Fig. 1. Error in the momentum determination for 5- and 20-Bev/c electrons, as a function of track length and radiation length X.